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Parameter Dependence of Ballistic Velocity in Deterministic Diffusion in the Form of Devil's Staircase

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1 Introduction

The diffusion coefficient defined via conventional mean square displacements is not sufficient for describing non-Gaussian statistics of velocity caused by ballistic motion in diffusive processes. In order to remedy this deficiency, we introduce large-deviation statistics also known as thermo-dynamical formalism.

2 Large-deviation statistics of velocity in diffusive dynamics

Let us briefly describe large-deviation statistics following the series of studies by Fujisaka and Inoue [1]. Consider a stationary time series of velocity u. The average over time interval T is given by this formula, $\overline{u}_T(t) = \frac{1}{T}\int_t^{t+T} u\{s\}\,ds$ which distributes when T is finite. When T is much larger than the correlation time of u, the distribution of coarse-grained u is assumed to be an exponential form $P_T(u) \propto \mathrm{e}^{-S(u)T}$. Here we can introduce the fluctuation spectrum $S(u)$ as $S(u) = -\lim_{T\to\infty} \frac{1}{T}\log P_T(u)$. When T is comparable to the correlation time, correlation cannot be ignored, so non-exponential or non-extensive statistics will be a problem, but here we do not discuss this point further. Let q be a real parameter. We introduce the generating function M_q of T by this definition. $M_q(T) \equiv \langle \mathrm{e}^{qT\overline{u}_T}\rangle = \int_{-\infty}^{\infty} P_T(u)\mathrm{e}^{qTu}\,du$ We can here also assume the exponential distribution and introduce a characteristic function $\phi(q)$ as $\phi(q) = \lim_{T\to\infty}\frac{1}{T}\log M_q(T)$. The Legendre transform holds between fluctuation spectrum $S(u)$ and characteristic function $\phi(q)$, which is obtained from saddle-point calculations: $\frac{dS(u)}{du} = q, \quad \phi(q) = -S(u(q)) + qu(q)$. In this transform a derivative $d\phi/dq$ appears, and it is a weighted average of $\overline{u}_T$, $u(q) = \frac{d\phi(q)}{dq} = \lim_{T\to\infty}\frac{\overline{u}_T\mathrm{e}^{qT\overline{u}_T}}{M_q(T)}$ so we find that q is a kind of weight index. We can also introduce susceptibility $\chi(q) = \frac{du(q)}{dq}$ as a weighted variance. These statistical structure functions $S(u), \phi(q), u(q), \chi(q)$ constitute the framework of statistical thermodynamics of temporal fluctuation, which characterize static properties of chaotic dynamics.

3 Conclusion

We deal with extracting the non-Gaussian characteristics of the phenomenon of diffusion. Then, we refer to the mapping system in which Klages and Dorfman discovered complex dependence of diffusion coefficients on the parameter [2]. A graph plotting the weighted mean velocity $u(q)$ of diffusive particles against parameter a is numerically obtained. This demonstrates that the graph $u(q \to \infty)$ corresponding to the velocity of the ballistic trajectory has a structure resembling the devil's staircase. It is found that the unstable periodic orbit corresponding to the ballistic motion with the largest velocity in this system changes in a complex manner depending on the value of parameter a. This seems to be one of the factors explaining why the graphs of $u(q)$ have complex structures.

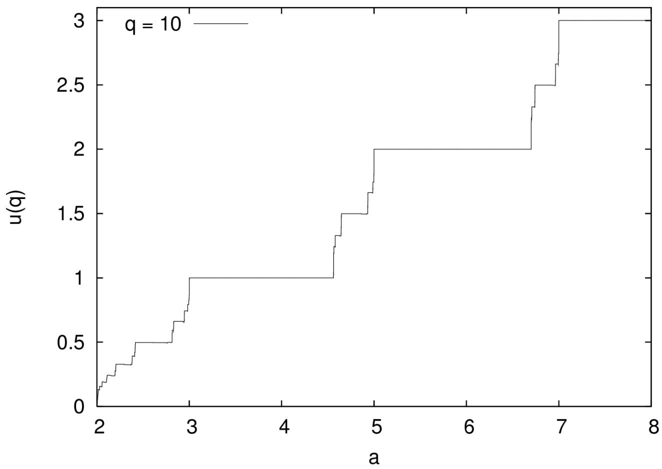

Fig.1: Parameter dependence of ballistic velocity

References

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[2] R. Klages and J. R. Dorfman, Phys. Rev. Lett. **74**, 387 (1995).