diffusion-fundamentals

The Open-Access Journal for the Basic Principles of Diffusion Theory, Experiment and Application

Anderson Localization and Generalized Diffusion

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1 Introduction

Disorder leads to important physical effects which are of quantum mechanical origin. This has been revealed by Anderson [1] in the study of a disordered tight-binding model. The problem has attracted great attention over many decades [2]. Subsequent to the ideas presented in our previous papers [3,4], we discuss here a new analytical approach to calculate the phase-diagram for the Anderson localization [1] in arbitrary spatial dimensions D. The transition from delocalized to localized states is treated as a *generalized diffusion* which manifests itself in the divergence of diagonal correlators. This divergence is controlled by the Lyapunov exponent γ which is the inverse of the localization length, $\xi=1/\gamma$.

2 Anderson localization as generalized diffusion

The Lyapunov exponent plays the role of an order parameter: $\gamma=0$ for the metallic phase and $\gamma\neq 0$ for the insulating phase. The value $\gamma\neq 0$ for the insulating phase implies a divergence of certain mean values as a function of the index $\mathbf{n}$. This divergence is a general property of a number of different stochastic systems. To give a clear picture we consider the equations for 1-D systems and in particular for Anderson localization (recursion relation for Schrödinger equation)

$$\Psi_{n+1}=(E-\varepsilon_n)\Psi_n-\Psi_{n-1} \qquad (1)$$

and diffusion (random walks)

$$\Psi_{n+1}=\Psi_n+\varepsilon_n \, . \qquad (2)$$

The ε_n are random variables to simulate the disorder in eq.(1) or random walks in eq.(2). The simplest and most important characteristic of diffusion is the second moment, $<\Psi^2_n>$ (diagonal correlator for 1-D). For symmetric diffusion with $<\varepsilon_n>=0$ and $<\varepsilon^2_n>=\sigma^2$

$$<\Psi^2_n> =\Psi^2_0+\sigma^2 n \, . \qquad (3)$$

To detect the diffusion, it is sufficient to demonstrate the divergence of the second moment of the amplitude and to establish its law of time-dependence. A divergence of this variable for $n\to\infty$ is an unambiguous proof for the existence of diffusion.

The equation (1) is nothing else but a generalization of equation (2). The mean value $<\Psi^2_n>$ $=\mathbf{f(n)}$ describes in this case generalized diffusion. If **f(n)** is bounded, **f(n)** $< \infty$, then the proper dynamics in equation (1) is stable and we have no diffusion (Lyapunov exponent γ**=0)**. This corresponds in a physical interpretation to the existence of delocalized states. A divergence of the function **f(n)** for $\mathbf{n \to \infty}$ corresponds to generalized diffusion (localized states); in this case one could distinguish in addition between non-exponential localization (**f(n)** is a non-exponential function) and exponential localization, $\mathbf{f(n) \propto \exp(2\gamma n)}$, with $\gamma \neq \mathbf{0}$.
The appearance of the generalized diffusion arises due to the instability of a fundamental mode corresponding to diagonal correlators. The generalized diffusion can be described in terms of signal theory, which operates with the concepts of input and output signals and the filter function [5]. Delocalized states correspond to bounded output signals, and localized states to unbounded output signals, respectively. Transition from bounded to unbounded signals is defined uniquely by the filter function. Although the filter is quite in general defined by an integral, it is possible for 2-D to give a representation in terms of radicals. As a consequence we have been able to find an exact analytical solution for the generalized Lyapunov exponents for the well-known and notorious Anderson problem in 2-D. In this way the phase diagram is obtained.
In the 1-D case all states are localized for arbitrarily small disorder in agreement with existing theories. In the 2-D case for larger energies and large disorder all states are localized, but for certain energies and small disorder, extended and localized states coexist.
Recent experiments [2] have challenged conventional wisdom (*all states in 2-D are localized*) about disordered 2-D systems. Ilani et al. [6] have studied spatial structures at the metal-insulator transition in 2-D. They found that there is a coexistence of two phases.

3 Conclusion

We have shown for 2-D [1] that, in principle, there is the possibility that the phase of delocalized states exists for a *non-interacting electron system*. For energies and disorder, where extended states may exist, we find a coexistence of these localized and extended states. Thus the Anderson metal-insulator transition exists and should be regarded as a first-order phase transition.

References

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