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Deterministic Chaos and Diffusion: From Theory to Experiments

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1 Introduction: deterministic chaos and diffusion

A century ago Einstein developed a theory of diffusion that is based on the assumption of stochasticity for a Brownian particle. On a microscopic level, however, the particle's dynamics is governed by Newton's deterministic equations of motion, which typically are highly nonlinear. This motivates to replace the hypothesis of stochasticity underlying traditional statistical mechanics by the hypothesis of microscopic deterministic chaos in the equations of motion of a Brownian particle. Such a deterministic chaos approach to transport takes the full memory of a particle into account while in stochastic models dynamical correlations tend to be neglected. One may thus ask whether for a given deterministic dynamical system there exist non-trivial dynamical correlations; if so, whether fundamental principles of transport theory such as Fick's laws are still recovered; or if not, what the precise consequences for diffusive transport are. These questions form the problem of deterministic chaos and diffusion.

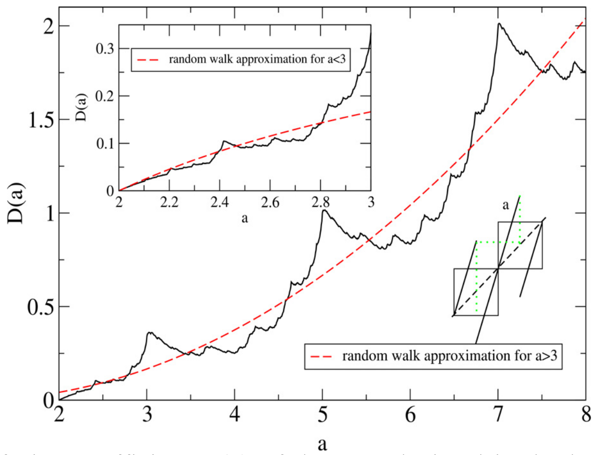

Figure 1: Diffusion coefficient $D(a)$ of the map depicted in the lower right corner. Shown are two pieces of the periodically continued map plus a trajectory segment of a diffusing particle. The slope a determining the average step lengths of a walker serves as a control parameter. The dashed lines represent random walk solutions for $D(a)$, the inset depicts a blowup for the initial region of the main graph.

2 From simple models to experiments

Deterministic versions of a random walk on the line provide simple models for which these questions can be answered. Here the `coin tossing' determining whether a walker makes one step to the left or to the right is replaced by the dynamics of a one-dimensional chaotic map. Defining such maps on a periodic lattice plus a deterministic coupling leads to dynamical systems exhibiting deterministic diffusion. An example is the piecewise linear map shown in Fig. 1, which depends on a control parameter. Exact results from a variety of theoretical methods confirm that its diffusion coefficient exists. Surprisingly, however, it is a fractal function of the control parameter while simple random walk theory predicts monotononicity [1]. This oscillatory structure originates from correlated forward- and backward scattering sequences that are unstable under parameter variation. Similar irregular diffusion coefficients are found in simulations of periodic Lorentz gases mimicking diffusion of an electron in a crystal [2,3]. They also exist in simulated diffusion of granular particles on oscillating corrugated floors [4]. Experiments on such vibratory conveyors, as they are called in industrial applications, indeed reveal irregular parameter dependencies for the mean velocity of these particles.

3. Conclusion

Microscopic deterministic chaos can generate transport coefficients that are irregular functions of control parameters. Theory predicts that this phenomenon is typical for low-dimensional dynamical systems exhibiting some periodicity. Examples are antidot lattices, where oscillations have already been measured experimentally, and Josephson junctions. Moving theory and experiments on irregular transport coefficients closer together poses an important open question.

References

[1] R. Klages, J.R. Dorfman, Phys. Rev. Lett. 74, 387 (1995)
[2] R. Klages, N. Korabel, J. Phys. A: Math. Gen. 35, 4823 (2002)
[3] T. Harayama, R. Klages, P. Gaspard, Phys. Rev. E 66, 026211 (2002)
[4] R. Klages, I.F. Barna, L. Matyas, Phys. Lett. A 333, 79 (2004)