Diffusion-wave inverse problem thermal conductivity depth-profile reconstructions using an integral equation approach

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The thermal-wave boundary-value problem in a semi-infinite thermophysically inhomogeneous solid:

$$\frac{\partial}{\partial x}\left[k(x)\frac{\partial T(x,t)}{\partial x}\right]=C(x)\frac{\partial T(x,t)}{\partial t}\ ;\quad t\geq 0\ ;\ 0\leq x<\infty\ ;\ \text{with} \tag{1a}$$

$$-k(0)\frac{\partial T(x,t)}{\partial x}\bigg|_{x=0}=\frac{1}{2}F_0 e^{i\omega_0 t}\quad \text{and}\quad C(x)\equiv\rho(x)c(x)\quad , \tag{1b}$$

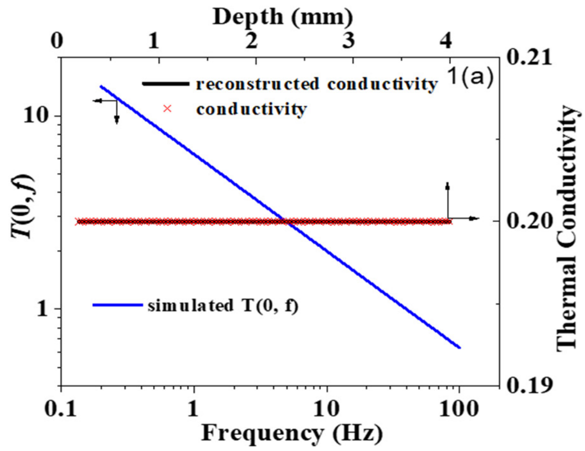
where F_0 is the incident (photo)thermal flux, $\rho(x)$ is the density, $k(x)$ is the thermal conductivity and $c(x)$ the specific heat of the material; ω_0 is the modulation angular frequency. Introducing the Green function, $G(x\,|\,x_0)$, approach yields the formal solution to the inhomogeneous problem in the frequency domain in the form of a Fredholm-type integral equation:

$$T(x;\omega_0)=-i\omega_0\int_0^\infty G(x|x_0)T(x_0;\omega_0)C(x_0)dx_0-k(0)G(x|0)\left[\frac{dT(x_0;\omega_0)}{dx_0}\bigg|_{x_0=0}\right] \tag{2}$$

Solving this equation results in an inverse relation between the thermal conductivity depth profile and the inverse Laplace transform of the thermal diffusion-wave frequency response:

$$\frac{1}{k(x)}=\frac{d}{dx}\left(L^{-1}\left[\frac{T(0;p)}{p}\right]\right);\ L^{-1}\ \text{denotes inverse spatial Laplace transformation.} \tag{3}$$

Figure (1a) shows inversion pairs (conductivity and frequency response) in the case of constant thermal conductivity, in agreement with the well-known analytical solution [A. Mandelis, *Diffusion-Wave Fields*, Springer, New York 2001, Chap. 2.1]. Figure (1b) shows the inversion pair in the case of an increasing $k(x)$ depth profile. The method is currently being applied to the non-destructive reconstruction of thermophysical depth profiles in manufactured solids such as metal powder compacts used for parts in the automotive industry.



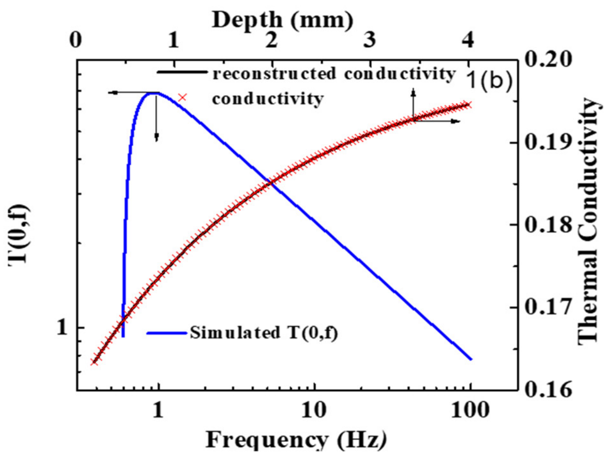


Figure 1: Conductivity and frequency response in case of constant thermal conductivity (a) and increasing conductivity depth profile.